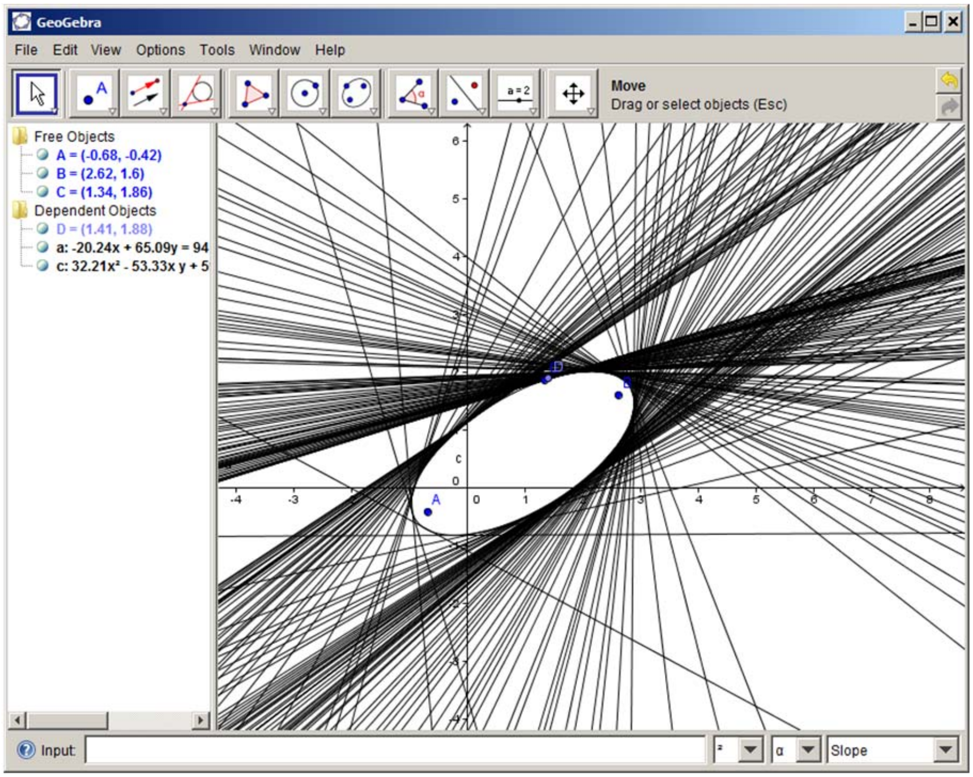

GeoGebra

Open Source Software Dynamically Linking Number, Algebra, and Geometry

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<u>GeoGebra: Linking Numeric, Algebraic and Geometric Representations</u>

The following instructions lead through steps that build linked numeric, graphic, algebraic, and geometric models for the flight of the ball in the images above.

Accessing GeoGebra

GeoGebra is open-source software and a copy can be downloaded on to your computer from:

http://www.geogebra.org/cms/en/download

The GeoGebra website (www.geogebra.org) also provides much support for users of the software.

GeoGebra is written in Java and can be run as a Web application if the software is not installed on the computer you are using. Go to http://www.geogebra.org/cms/en/download and click on 

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Getting the Files Needed for this Workshop

Open a Web browser

Go to: **http://queensgeogebra.pbworks.com/**

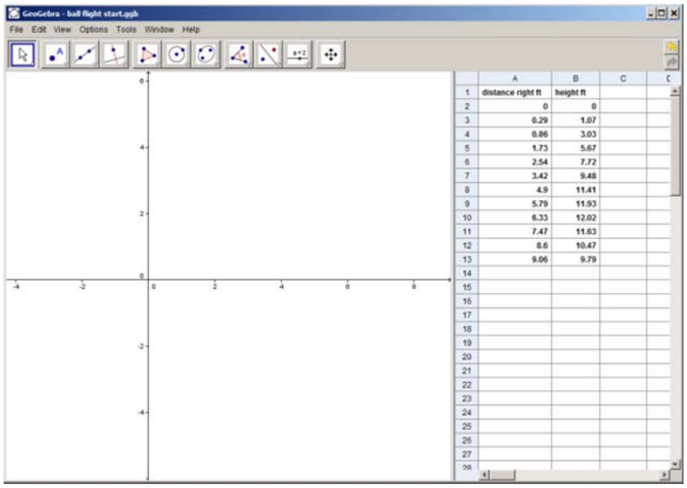
Click: **<u>Open Source Software Dynamically Linking Number, Algebra, and Geometry</u>**

A Numeric Representation of the Ball's Flight

Click: **<u>ball flight start.ggb</u>** and **Open** if you have GeoGebra on your computer

or

Click: **<u>ball flight start.html</u>** if GeoGebra is not installed on your computer.



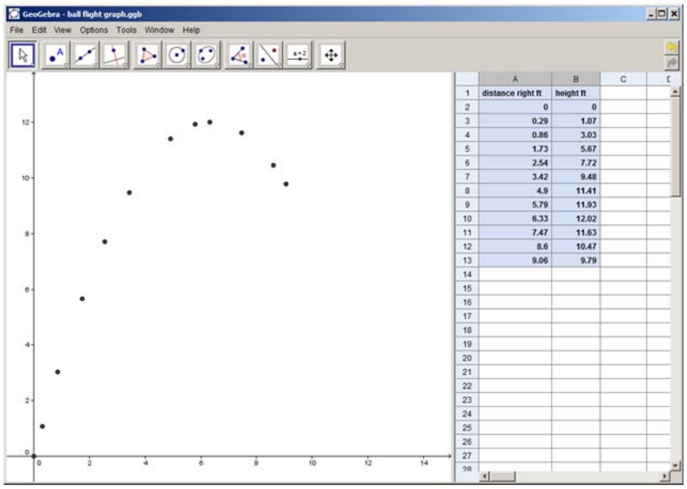
The Spreadsheet View in GeoGebra provides an Excel-like spreadsheet. The numbers in columns A and B record the distances in feet, right and up from the point on the floor occupied by the ball in the first image (left end of the folded bleachers).

Graph Representation

Move the cursor to the graph pane and adjust the axes by:

Right-click > **Graphics View** > **Axes** > **x** and set x axis **-1 to 15**
> **y** and set y axis **-1 to 15** and set **xAxis : yAxis = 1:1**

Highlight the A and B columns of the spreadsheet:
Hold shift-key and click on each column
right-click > **Create List of Points**



The spreadsheet values and points are linked. Add 1.0 to any of the A column values and note the effect on the related point. Restore the A value by subtracting 1.

What type of curve might pass through the points?

Algebraic Representation: Equation in Standard Form: $y=ax^2+bx+c$

Click: **View** > **Algebra View** (to show Algebra window)

Click: **View** > **Input Bar**

In the Input Bar type **y=-3x^2** and **Enter**

This is obviously not the correct equation. We will adjust and attempt to make the curve fit the points.

In the Free Objects list double-click: $y=-3x^2$

Modify the equation and check fit by **Enter**. Repeat to improve the fit.

A simpler approach to modifying the equation

In the Input Bar type:
a=1 Enter
b=1 Enter
c=1 Enter
y=a*x^2+b*x+c Enter

This puts the parameters a, b, and c in the Free Objects list and the equation $y=x^2+x+1$ or $y=ax^2+bx+c$ in the Dependent Objects list.

Click (highlight): **a=1** and use the up and down cursor arrow keys to adjust the parameter, a

Using this approach with all three parameters adjust the curve to pass through the points.

Alternatively:

Right-click a > **click Show Object**
Use the slider for a to adjust the value of the a parameter

Repeat for b and c

Use the sliders to adjust the values of all three parameters and adjust the curve to pass through the points. If the range of values is not sufficient:

Right-click on the slider > **click Object Properties…**
Adjust the minimum and/or maximum values

We now have linked numeric (spreadsheet) and algebraic (equation & graph) representations.

Algebraic Representation: Function in Vertex Form: $f(x)=a(x-h)^2+k$

If you would like to retain an unexpanded form for the expression in x then function notation is required.

Create two new sliders for parameters h and k.

Click on the slider tool  and select **Slider**

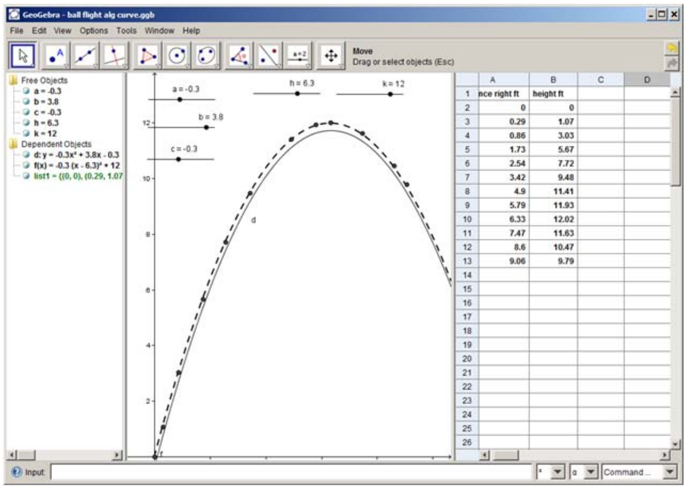
Click on the graph window where you want the slider. Change the label to 'h', and **Apply**.

Repeat to get a slider for parameter 'k'.

We now have two more Free Objects h and k to use in a function

In the Input Bar type **f(x)=a*(x-h)^2+k** and **Enter**

Use the sliders for a, h, and k to adjust this new curve to pass through the points.



Reversing the Algebraic and Numeric Linking

Put a point on the curve for the function f(x)

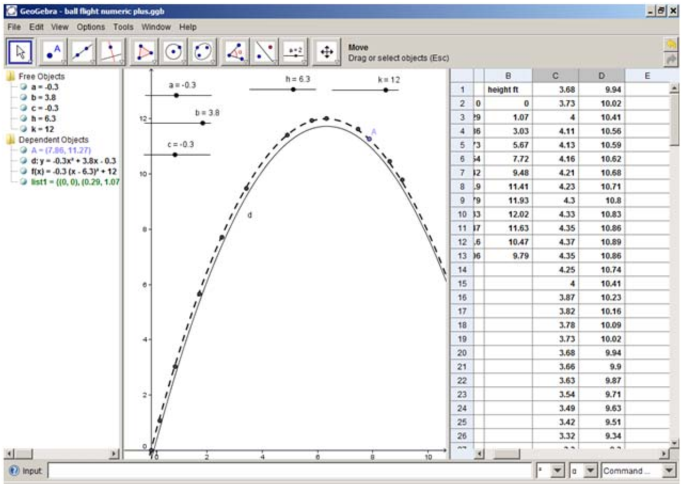
Click: **New Point** 

Click on the curve

Click the **Move** tool 

Right-click on the point and select **Trace to Spreadsheet**. Click and drag the point.

The algebraic (equation & graph) and numeric (spreadsheet) representations are linked.



Building a Geometric (Conic) Model

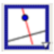
Click: **View** > **Spreadsheet** (to hide Spreadsheet window and create space)

We will use the Euclidean focus-directrix construction for a parabola. A parabola is the curve traced out by a point moving so that it is equidistant from a fixed point (focus) and a fixed line (directrix). (see the definition at: http://www.mathwords.com/d/directrix_parabola.htm)

Directrix:

Put a point on y-axis at approximately y=13: Click: **New Point** > Click on **y-axis**

Construct a line through this new point perpendicular to y-axis. Note the steps are opposite to GSP – select the type of construction then the objects.

Select: **Perpendicular Line**  and click the **new point** and the **y-axis**

Focus:

Place another new point below the perpendicular (horizontal) line

Construct the Parabola:

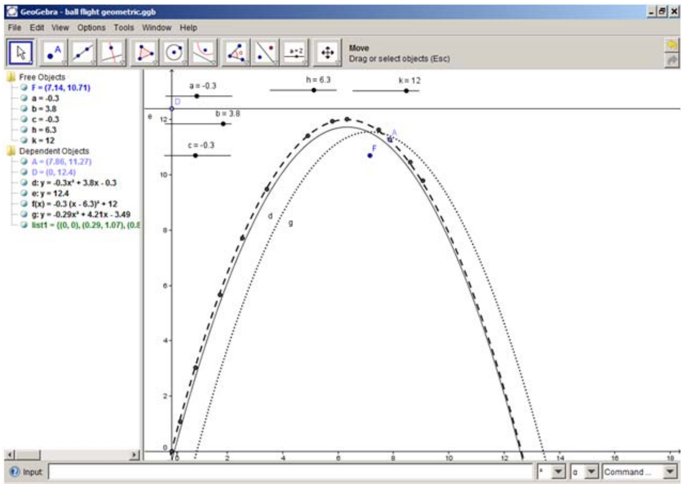
Click the **Conic** option  and select **Parabola** 

Click: the **directrix** (horizontal line) and **focus** (new point below the directrix)

Using the **Move** tool, adjust the locations of the focus and directrix to create a parabola that passes through the points.

Select the parabola and right-click. Select **Equation y=ax^2+bx+c**

The geometric (parabola) and algebraic (equation) representations are now linked.



Using the Algebraic Representation

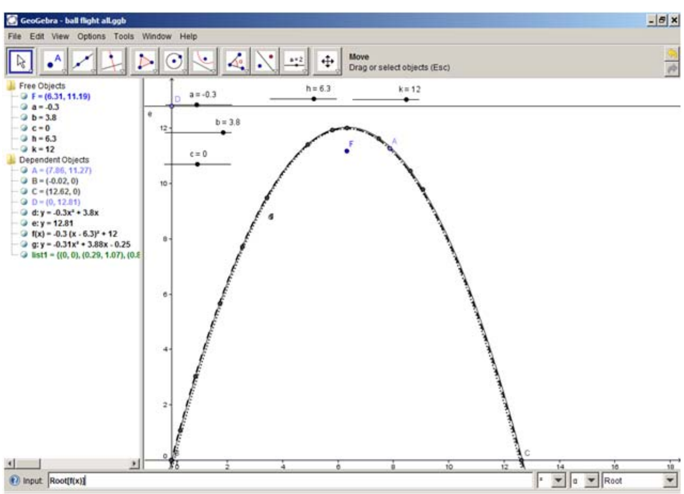
We could use any of the models to determine the point where the ball would have hit the floor a second time if there was no net, but the function representation, f(x), provides a direct route to calculating the x-coordinate or distance to the right.

Click **Command…**

Scroll down the list of commands and select **Root**

Adjust the contents of the Input Bar to look read **Root[f(x)]** and **Enter**

In the Dependent Objects, read the x-coordinate of the new root point.



Adding Images to Connect to the Problem Context

You can add images (gif, jpeg, jpg, tif, png, bmp) to GeoGebra constructions using the **Insert Image** tool. The image may be set as background, pegged to the axes system, or may be linked to any point on the graphics plane. With this we can connect the ‘quadratic function – parabola’ activity to the original basketball context. When live, the picture of the basketball below can be moved along the parabola.